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The statistical mechanical interpretation of algorithmic information theory (AIT, for short) was developed in our first work [K. Tadaki, Local Proceedings of CiE 2008, pp. 425–434, 2008] on this subject by introducing into AIT the notion of thermodynamic quantities at temperature T , such as partition function $Z_U(T)$, free energy $F_U(T)$, energy $E_U(T)$, and statistical mechanical entropy $S_U(T)$. These quantities are real functions of real argument $T > 0$ defined based on the halting set of an optimal computer U , which is a universal decoding algorithm used to define the notion of program-size complexity. We discovered that, in the interpretation, the temperature T equals to the partial randomness of the values of all these thermodynamic quantities, where the notion of partial randomness is a stronger representation of the compression rate by means of program-size complexity. Furthermore, we showed that this situation holds for the temperature itself as a thermodynamic quantity, namely, the computability of the value of partition function $Z_U(T)$ gives a sufficient condition for $T \in (0, 1)$ to be a fixed point on partial randomness.

In our second work [K. Tadaki, Proceedings of LFCS'09, Springer's LNCS, vol. 5407, pp. 422–440, 2009], we showed that the computability of each of the thermodynamic quantities $F_U(T)$, $E_U(T)$, and $S_U(T)$ also gives the sufficient condition, in addition to $Z_U(T)$. Moreover, based on the statistical mechanical relation $F_U(T) = -T \log_2 Z_U(T)$, we showed that the computability of $F_U(T)$ gives completely different fixed points from the computability of $Z_U(T)$.

In this talk, we develop the statistical mechanical interpretation of AIT further and pursue its formal correspondence to normal statistical mechanics. In particular, we unlock the properties of the sufficient conditions further by introducing the notion of the composition of systems into AIT, which corresponds to that in normal statistical mechanics. As a result, we obtain the following theorem.

THEOREM 1. *There exists a recursive infinite sequence U_1, U_2, U_3, \dots of optimal computers such that (i) $\mathcal{Z}(U_k) \cap \mathcal{Z}(U_l) = \emptyset$ for any distinct k and l , where $\mathcal{Z}(U_k) = \{T \in (0, 1) \mid Z_{U_k}(T) \text{ is computable}\}$, and (ii) For every k , if $T \in \mathcal{Z}(U_k)$, then $Tn \leq H(T \upharpoonright_n) + O(1)$ and $H(T \upharpoonright_n) \leq Tn + o(n)$, and therefore $\lim_{n \rightarrow \infty} H(T \upharpoonright_n)/n = T$, where $T \upharpoonright_n$ is the first n bits of the base-two expansion of T .*

In addition, we can show that the same theorem holds for each of the thermodynamic quantities $F_U(T)$, $E_U(T)$, and $S_U(T)$ with the same recursive infinite sequence U_1, U_2, U_3, \dots .

Key words: algorithmic information theory, algorithmic randomness, statistical mechanics, thermodynamic quantities, fixed point theorem, partial randomness, composite system